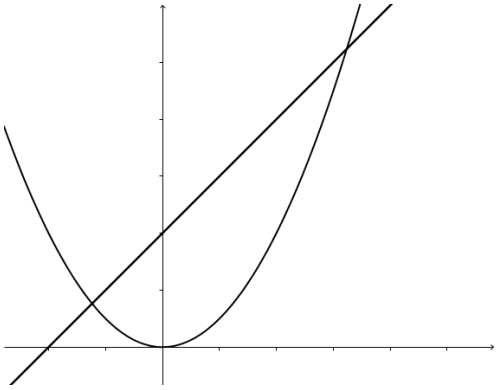
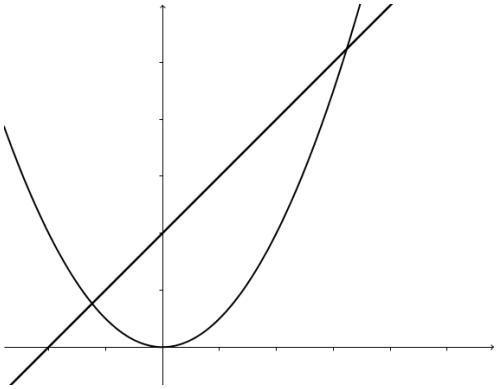
The Area Between Two Curves

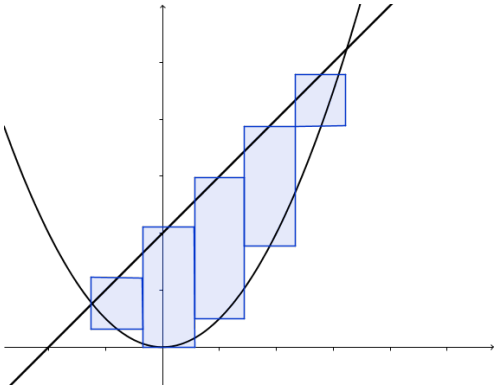
Introduction

- **Intuitive Definition**



- **Precise Definition**





The Area Between Two Curves

If f and g are continuous and $f(x) \geq g(x)$ over the interval $[a, b]$, then the **area bounded** by $y = f(x)$ and $y = g(x)$ and the lines $x = a$, $x = b$ is given exactly by

$$A = \int_a^b (f(x) - g(x))\ dx$$

Example 1.

Find the area of the region bounded above by $y = e^x$, bounded below by $y = x$, and bounded on the sides by $x = 0$ and $x = 1$.

Example 2.

Find the area bounded by the graphs of

$$f(x) = \frac{1}{2}x + 3 \qquad g(x) = -x^2 + 1 \qquad x = -2 \qquad x = 1.$$

Example 3.

Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

What can happen at intersection points?

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$$A = \int_a^b |f(x) - g(x)| \, dx$$

How to Compute the Area of the Region Between Two Curves

1. Locate any intersection points.
2. Identify the Top and Bottom Functions on each sub-interval.
3. Sketch the region.
4. Setup the integral and evaluate.

Example 4.

Find the area bounded by the graphs of

$$f(x) = x^2 - x \qquad g(x) = 2x \qquad \text{for } -2 \leq x \leq 3$$

Example 5.

Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \dfrac{\pi}{2}$.

What if the top and bottom curves are given by the same equation?

Example 6.

Find the area of the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.